

**Study on Adaptive Inverse Dynamics Control Scheme for A
Two Compartment Lung System**

A Thesis

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By

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Abstract

A study of using an adaptive inverse dynamics control technique to a two-compartment modeled respiratory system. Based on the nonlinear respiratory model and desired respiratory volumes, the adaptive inverse dynamics control scheme consisting of a control law and an adaptation law is then applied. The control law has the structure of the two-compartment inverse dynamical model but uses estimates of the dynamics parameters in the computation of pressure applied to the lungs. The adaptation law uses the tracking error to compute the parameter estimates for the control law. The preliminary results indicate that the tracking errors can be improved if the parameter values associated with the adaptation law are properly chosen, and the performance is also robust despite relatively large deviations in the initial estimates of the system parameters.

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Chapter 1

Introduction

1.1 Respiratory System

1.1.1 Respiration

Respiration is the trading of O_2 and carbon dioxide between the environment and the body cells. In people, this procedure incorporates spark and termination, dispersion of O_2 from alveoli to the blood and of carbon dioxide from the blood to the alveoli, and the vehicle of O_2 to and carbon dioxide from the body cells by method for the circulatory framework. Breath is crucial to life as it supplies fuel (counting O_2) and evacuates waste items (counting carbon dioxide) from the cells.

The body has moderately little limit for capacity of the respiratory gasses so the respiratory control framework must endeavor to supply O_2 and uproot carbon dioxide at the rate at which the cells require. This segment manages the essential standards included in respiratory control, baby breath and the effect that changing body temperatures and warm control have on respiratory control.

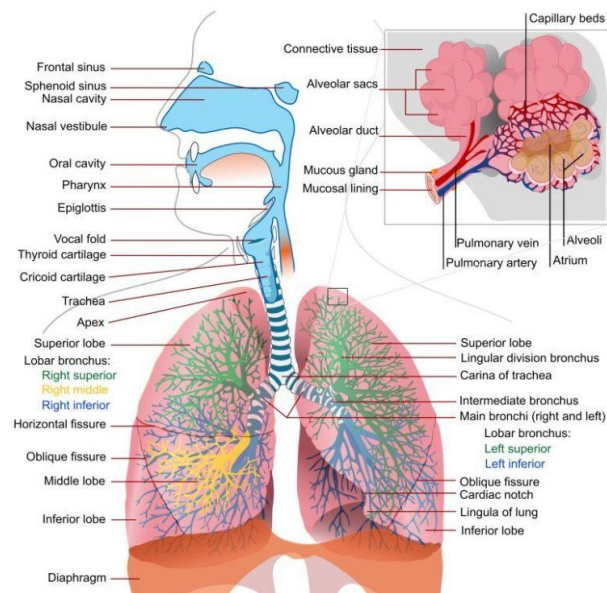


Fig 1 : Respiratory Model

1.1.2 Inhalation

Inhalation is launched by the stomach and backed by the outer intercostal muscles. Typical resting breaths are 10 to 18 breaths every moment, with a period time of 2 seconds. Amid vivacious inward breath (at rates surpassing 35 breaths every moment), or in approaching respiratory disappointment, embellishment muscles of breath are enlisted for backing.

Under ordinary conditions, the stomach is the essential driver of inward breath. At the point when the stomach gets, the ribcage extends and the substance of the guts are moved descending. This outcome is a bigger thoracic volume and negative weight (concerning barometrical weight) inside the thorax. As the weight in the midsection falls, air moves into the directing zone. Here, the air is sifted, warmed, and humidified as it streams to the lungs.

1.1.3 Exhalation

Exhalation is a detached methodology; notwithstanding, dynamics or constrained exhalation is accomplished by the stomach and the inside intercostal muscles. Amid this procedure air is constrained or breathed out. The lungs have a characteristic flexibility, as they force from the stretch of inward breath; wind currents pull out until the weights in the midsection and the environment achieve harmony. Amid constrained exhalation, as when extinguishing a candle, expiratory muscles including the muscular strength and inward intercostal muscles create stomach and thoracic weight, which powers let some circulation into of the lungs.

1.1.4 Gas exchange

The significant capacity of the respiratory framework is gas trade between the outside environment and a life form's circulatory framework. In people and different well evolved creatures, this trade encourages roation of the blood with an associative evacuation of carbon dioxide and different vaporous metabolic squanders from the dissemination. As gas trade happens, the corrosive base equalization of the body is kept up as a major aspect of homeostasis. On the off chance that legitimate ventilation is not kept up, two contradicting conditions could happen: respiratory acidosis, an existence undermining condition, and respiratory alkalosis.

Upon inward breath, gas trade happens at the alveoli, the minor sacs which are the essential practical segment of the lungs. The alveolar dividers are to a great degree flimsy (approx. 0.2 micrometers). These dividers are made from a solitary layer of epithelial cells near to the aspiratory vessels which are made from a solitary layer of endothelial cells. The nearby vicinity of these two cell sorts permits porousness to gasses and, thus, gas trade. This entire instrument of gas trade is conveyed by the straightforward sensation of weight distinction. At the point when the pneumatic force is high inside the lungs, the air from lungs stream out. At the point when the pneumatic stress is low inside, then wind streams into the lungs.

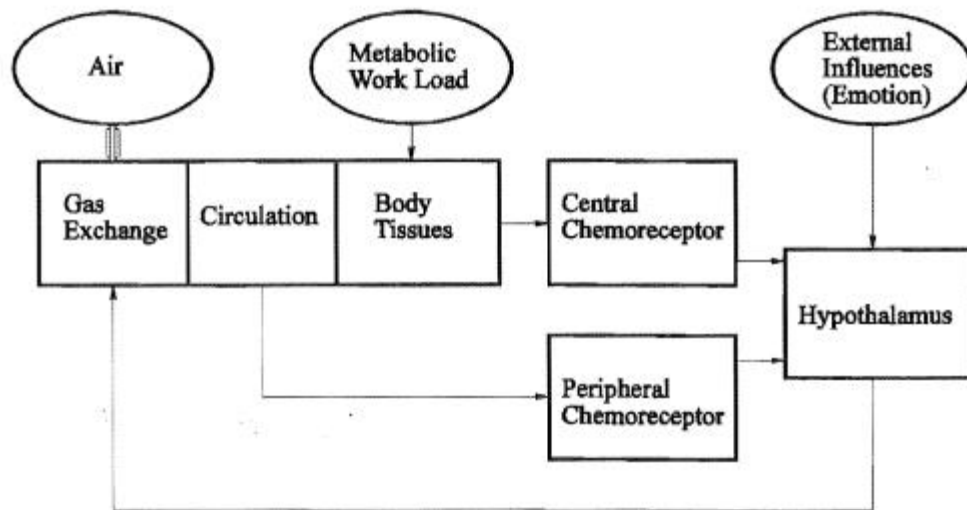


Fig 1.2: Schematic diagram of the human respiratory control system.

Above figure shows the fundamental standards of the respiratory control framework. O_2 noticeable all around is brought to, and carbon dioxide is expelled from the body cells through the lungs and the circulatory framework. The higher the ventilation rates of the lungs, the more prominent the amount of gas that is pumped into and out of the circling blood. The body cells change the gas fractional weights in the blood as they devour O_2 and produce carbon dioxide at a rate relative to the cell metabolic rate. This is reliant on variables, for example, practice or hotness creation necessities of the body.

Chemoreceptor structures give data on the O_2 and carbon dioxide incomplete weights in the blood and the carbon dioxide gas fractional weight in the cerebrospinal liquid (CSF) of the cerebrum. This data is passed on by afferent neural pathways to the respiratory control focus in the hypothalamus of the cerebrum, which modifies the ventilation of the lungs to keep up gas fractional weights at typical qualities. To a constrained degree respiratory gasses are released in the body. This gives a cradle to sudden unsettling influences to the gas fractional weights in the body which may happen, for instance, as an aftereffect of quick changes in the levels of activity.

Carbon dioxide is principally released in the body tissues and just a little amount is released in arrangement in the blood. On the other hand, little O_2 is released in the body tissues yet noteworthy amounts are released in synthetic mix with hemoglobin in the blood. Albeit respiratory control structures a complete framework, it can't be viewed as autonomous of different frameworks inside the body. One framework that has a noteworthy bearing on breath is the cardiovascular framework. Should the metabolic rate of a specific organ build then, so as to encourage the extra requests put on the vehicle of metabolites to and from that organ, an increment in blood stream to that organ should likewise happen. This change in blood stream adjusts the progress of the respiratory framework.

Chapter 2

Multi-compartment Respiratory System

The lungs are especially powerless against intense discriminating diseases. Respiratory failure can come about not just from essential lung pathology, for example, pneumonia, additionally as an auxiliary result of heart disappointment or provocative ailment, for example, sepsis or trauma. At the point when this happens, it is fundamental to backing patients while the principal malady methodology is addressed. For illustration, a patient with pneumonia may require mechanical ventilation while the pneumonia is being dealt with anti-toxins, which will in the end adequately cure the disease. Since the lungs are vulnerable against discriminating disease and respiratory failure is regular, backing of patients with mechanical ventilation is exceptionally basic in the concentrated consideration unit.

The objective of mechanical ventilation is to guarantee sufficient ventilation, which includes a greatness of gas trade that is prompts the coveted blood level of carbon dioxide (CO_2), and satisfactory rotation, which includes a blood fixation of O_2 that will guarantee organ capacity. Accomplishing these objectives is muddled by the way that mechanical ventilation can cause intense lung damage, either by expanding the lungs to inordinate volumes or by utilizing over the top weights to blow up the lungs. The test to mechanical ventilation is to produce the wanted blood levels of CO_2 and O_2 without bringing about additional intense lung harm. The soonest essential modes of ventilation can be ordered, nearly, as volume controlled alternately weight controlled.

In volume-controlled ventilation, the lungs are expanded (by the mechanical ventilator) to a predefined volume and after that permitted to inactively collapse to the gauge volume. With the expanding accessibility of microchip innovation, it has been conceivable to plan mechanical ventilators that have control calculations which are more complex than basic volume or weight control. Cases are relative aid ventilation, versatile bolster ventilation, Brilliant Consideration ventilation, and neutrally balanced ventilation. In corresponding aid ventilation, the ventilator measures the quiet's volume and rate of inspiratory gas stream, and afterward applies weight bolster in extent to the quiet's inspiratory exertion.

In this mode of ventilation, roused O_2 what's more, positive end-expiratory weight is physically balanced by the clinician. In versatile bolster ventilation, tidal volume and respiratory rate are naturally balanced.

The quiet's respiratory example is measured point astute in time and bolstered back to the controller to give the obliged (target) tidal volume furthermore, persistent respiratory rate. Versatile bolster ventilation does not give nonstop control of moment ventilation, positive end-expiratory weight, and enlivened O_2 ; these parameters need to be balanced physically.

2.1 One-compartment and Two-compartment Linear Respiratory system

The test to mechanical ventilation is to create the sought blood levels of carbon dioxide and O_2 without creating additional intense lung damage. With the expanding accessibility of micro-chip innovation, it has been conceivable to outline incompletely mechanized mechanical ventilators with control calculations or giving volume or weight. The mechanical properties of the respiratory system are for inferred from estimations of weight and now at the air· way opening. Customarily, these estimations have been connected through a single compartment model of the respiratory framework. As of late, then again, there has been significant enthusiasm for demonstrating low recurrence respiratory mechanics regarding two compartments, since this gives a greatly improved portrayal of trial information.

One sort of model records for local ventilation in homogeneities, genetic in the lung as far as two alveolar compartments. The other kind of model considers aspiratory ventilation to be homogeneous, while the tissues of the respiratory framework are displayed as being viscoelastic. In ordinary puppies, the fitting two-compartment model has been demonstrated to be the viscoelastic model. Because strange physiology, nonetheless, one must invoke a model having both viscoelastic tissues and ventilation in homogeneities. Extra trial information is needed to recognize such a model, and to evaluate these two phenomena. The procedure called converse displaying comprises first of concocting a model structure. The parameters of the model are then assessed to make the conduct of the model match an arrangement of test information as nearly as possible.

The parts of the model and their parameters ought to have sensible physiological partners. The little number of respiratory variables that can be measured (streams, volumes and weights, with a few refinements in regards to the level of their estimations) sets a point of confinement to the complexity of the models utilized and to the physiological translations that can be gotten from them picking between these two groups of models requires more data than is accessible in the connections between the amounts that are normally measured, in particular tracheal or trans pneumonic weight and tracheal flow. Although a solitary compartment model has been, and still is, broadly utilized as a part of respiratory physiology, two-compartment models give off an impression of being significantly more suitable in different circumstances.

There are two physiologically unmistakable classes of two-compartment models of enthusiasm for respiratory mechanics, those considering gas redistribution between diverse lung districts and those considering inherent tissue properties. This model has gotten to be popular to the point that the mathematical statement administering its conduct is for the most part (and mistakenly) alluded to as the "mathematical statement of movement of the respiratory mechanics", as opposed to as the "comparison of movement of a solitary compartment straight model of the respiratory mechanics".

This mathematical statement is

$$P(t) = RV(t) + EV(t) \tag{1}$$

where P is weight (as a rule aviation route opening weight or trans pulmonary weight), R is the resistance of the funnel to gas stream (V), E is the Elastance of the inflatable, V is the volume of the blow up over its casual volume, and t is time. Eq. 1 epitomizes various suppositions. Essential among these is that the respiratory framework carries on straightly, that will be that R is independent of V and E is free of volume. Another suspicion is that latency does not assume a huge part. This hypothesize is likely legitimate inside the scope of physiological breathing frequencies up to 2 Hz [8] and will be acknowledged in whatever remains of this paper

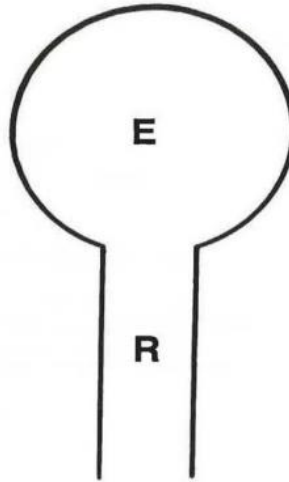


Fig 2.1: The single-compartment linear model of respiratory mechanics.

During volume cycling, qualities for E and R can be found by fitting equation 1 to estimations of P, V and \dot{V} utilizing various straight relapse or a related system, for example, the electrical subtraction strategy. Instinct recommends that a more detailed model than the one represented by equation above ought to give a superior portrayal of respiratory mechanical information. There are two general ways to deal with expanding the multifaceted nature of a model keeping in mind the end goal to more precisely portray an arrangement of information. One is to build the quantity of mechanical degrees of opportunity, that is, add more compartments the other is to make the current components of the model nonlinear, for example, by including a standard term to the parameter representing the airway resistance. The proper methodology relies on upon the information in question for case; a move which includes changing stream over a wide range may draw out the nonlinear impacts of a stream subordinate resistance, while a move which includes a scope of distinctive swaying frequencies at the same tidal volume may create conduct of an overwhelmingly multi-compartment nature. There is impressive test proof indicating the need of more than one compartment for depicting respiratory framework mechanics at low frequencies.

A model of two or more compartments is needed, with the compartments having distinctive time-constants given by the degrees of the compartmental resistances to elastane's. It has additionally been seen in canines that casual termination is not decently

depicted by a solitary exponential capacity, as Comparison 1 would foresee, however is greatly decently fitted by a twofold exponential capacity.

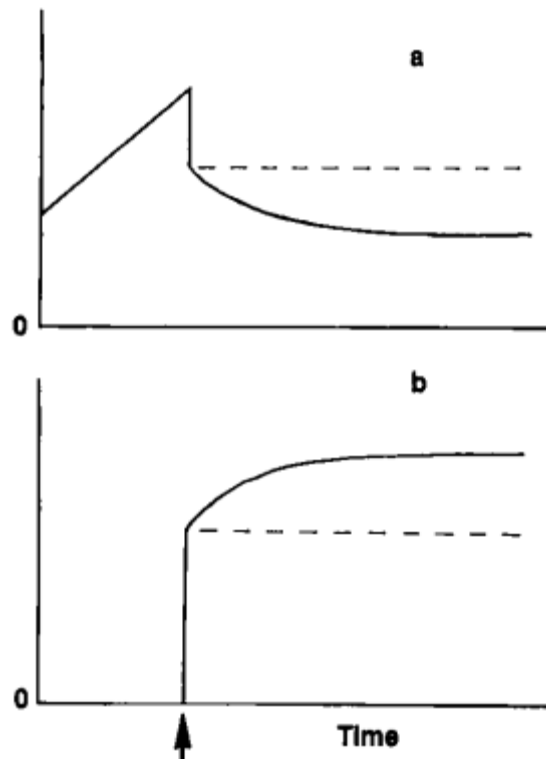


Fig 2.2: Time-constants of the compartmental resistances

Here again, one is constrained to conjure a two-compartment model with a quick and a moderate compartment. Comparative conclusions can be drawn from the time course of weight (tracheal, trans pulmonary or esophageal weight) that is seen after an end-inspiratory impediment. On the off chance that the respiratory framework acted as a solitary compartment direct model, the weight ought to promptly drop to its static quality upon stream interference and stay settled from there on. Really, stream intrusion brings about a sudden drop in weight took after by a further moderate decay towards the static quality stream intrusion amid close results in the inverse impact with a beginning fast hop in weight being trailed by a further moderate rise. Only a model including a quick and moderate compartment can represent this sort of conduct as delineated in Fig 2.2.

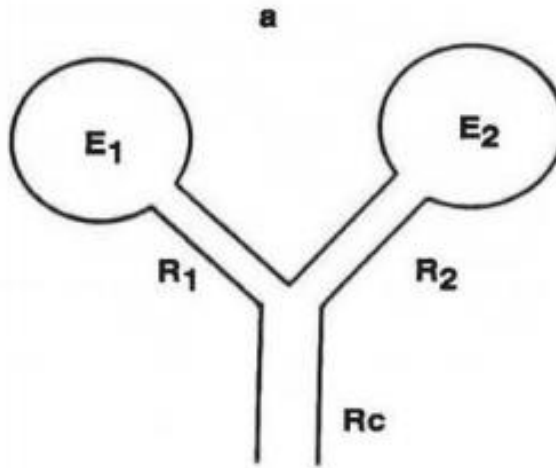


Fig 2.3: Linear two-compartment model

Linear two-compartment models of the respiratory system can be divided into two physiologically distinct types. One of these types, the gas redistribution model, ascribes the multi-compartment nature of the respiratory system to unevenness of gas distribution throughout the lungs. Two varieties of gas redistribution model have been proposed. The first, which we will call the parallel gas redistribution model, has dominated the literature since its introduction [8] and consists of a parallel arrangement of alveolar compartments connected by separate airways to the trachea (Fig 2.3).

2.2 Non-Linear one and two compartment systems

Respiratory failure, the lacking trade of carbon dioxide and O₂ by the lungs, is a typical clinical issue in basic consideration drug, and patients with respiratory failure oftentimes oblige support with mechanical ventilation while the hidden reason is distinguished and treated. The objective of mechanical ventilation is to guarantee satisfactory ventilation, which includes a greatness of gas trade that prompts the sought blood level of carbon dioxide, and sufficient O₂ation, which includes a blood centralization of O₂ that will guarantee organ capacity. Attaining to these objectives is confounded by the way that mechanical ventilation can cause intense lung harm, either by swelling the lungs to

inordinate volumes or by utilizing extreme weights to swell the lungs. The test to mechanical ventilation is to create the sought blood levels of carbon dioxide and O₂ without bringing on additional intense lung damage.

With the expanding accessibility of microchip innovation, it has been conceivable to plan in part robotized mechanical ventilators with control calculations for giving volume or weight control. More refined completely robotized model reference versatile control calculations for mechanical ventilation have likewise been as of late created. These calculations oblige a reference model for distinguishing a clinically conceivable breathing example. In any case, the respiratory lung models that have been introduced in the therapeutic and experimental writing have regularly expected homogenous lung capacity. Case in point, in similarity to a straightforward electrical circuit, the most well-known model has expected that the lungs be a solitary compartment described by its agreeability (the degree of compartment volume to weight) and the imperviousness to wind stream into the compartment. While a couple of specialists have considered two compartment models, mirroring the way that there are two lungs (right and left), there has been minimal enthusiasm for more models.

Early take a shot at the optimality of respiratory control systems utilizing basic homogenous lung models managed the recurrence of relaxing. Specifically, the creation anticipated the recurrence of breathing by utilizing a base work-rate basis. This work includes a static improvement issue and expects that the wind current example is an altered sinusoidal capacity, the created optimality criteria for the forecast of the respiratory wind stream design with settled inspiratory and expiratory periods of a breathing cycle. These outcomes were reached out in by considering a two-level progressive model for the control of breathing, in which the more elevated amount standard decides values for the general control variables of the ideal wind stream example got from the lower-level criteria, and the lower-level criteria focus the wind current example with the respiratory parameters picked by minimizing the more elevated amount basis.

Although the issue for distinguishing ideal respiratory examples has been tended to in the writing, the models on which these respiratory control instruments have been

distinguished are predicated on a solitary compartment lung model with consistent respiratory parameters. Then again, the lungs, particularly unhealthy lungs, are heterogeneous, both practically and anatomically, and are involved numerous subunits, or compartments, that contrast in their abilities for gas trade. Sensible models ought to consider this heterogeneity. Also, the imperviousness to gas stream and the consistence of the lung units are not consistent but instead shift with lung volume. This is especially valid for consistence. While more complex models involve more prominent many-sided quality, since the models are promptly introduced in the connection of dynamical frameworks hypothesis, advanced scientific apparatuses can be connected to their examination. Compartmental lung models are depicted by a state vector; whose segments are the volumes of the individual compartments.

A key question that emerges in the thought of nonlinear multi-compartment models is whether trial information bolsters a complex model. This inquiry can be tended to by considering a relationship to pharmacokinetics. The most punctual pharmacokinetic models were commonly one-compartment models. This mirrored the difficulties of testing and medication examine. These models were satisfactory for measuring medication attitude on quite a while scale. Case in point, straightforward one-compartment models were sufficient in depicting the aggregate leeway or volume of dissemination. On the other hand, for even open-circle control of medication focuses, the one compartment model was insufficient. More intricate models (two- and three-compartment models) were required that represented dispersion and additionally end methods.

So also, for versatile control of mechanical ventilation, that is, more exceptional controller architectures than basic volume- or weight controlled ventilation, more expound models are required, particularly when representing nonlinear agreeability and resistance and lung heterogeneity. Because pharmacokinetics, the control calculation must be as mind boggling as the information bolsters. This is additionally valid for control of mechanical ventilation. Stream and weight designs in the aviation route are not basic waveforms, although clinicians to date have demonstrated them thusly. There is extensive data installed in these waveforms. It is a simple errand to rearrange this system

to be consistent with the granularity of the information. The opposite methodology, notwithstanding, is unrealistic without the improvement of a general structure.

In this thesis, we augment the work to create ideal respiratory wind current examples utilizing a nonlinear multi-compartment model for a lung mechanics framework. (The use of the word ideal all through the thesis alludes to an ideal arrangement of the analytics of varieties issues tended to in the thesis and not an ideal breathing example in the feeling of respiratory physiology) First, we amplify the straight multi-compartment lung model given in to address framework model nonlinearities. Secondly, we broaden the execution functional grew in for the inspiratory and expiratory breathing cycles to infer an ideal wind current example utilizing established analytics of varieties strategies. Specifically, the physiological elucidation of the optimality criteria includes the minimization of work of breathing and lung volume quickening for the inspiratory breathing stage, and the minimization of the versatile potential vitality and fast wind stream rate changes for the expiratory breathing stage. Finally, we numerically incorporate the subsequent nonlinear two-point limit esteem issues to focus the ideal wind current examples over the inspiratory and expiratory breath.

The notation used here is standard. Specifically, \mathbb{R}^n denotes the set of $n \times 1$ real column vectors, and $\mathbb{R}^{n \times m}$ denotes the set of $n \times m$ real matrices. For $x \in \mathbb{R}^n$ we write $x \geq 0$ (resp., $x \gg 0$) to indicate that every component of x is non-negative. In this case, we say that x is a positive, respectively. Likewise, $A \in \mathbb{R}^{n \times m}$ is positive if every entry of A is positive. Furthermore, \mathbb{R}^n_+ and \mathbb{R}^n_{++} denote the nonnegative and positive orthants of \mathbb{R}^n , that is, if $x \in \mathbb{R}^n$, then $x \in \mathbb{R}^n_+$ are equivalent, respectively, to $x \geq 0$ and $x \gg 0$.

In this area, we develop the direct multi-compartment lung model of [6] to add to a nonlinear model for the element conduct of a multi-compartment respiratory framework because of a discretionary connected inspiratory weight. Here, we accept that the bronchial tree has a dichotomy building design, that is in every aviation route unit branches into two aviation route units of the ensuing era. What's more, we accept that the lung consistence is a nonlinear capacity of lung volume.

First, for simplicity of exposition, we consider a single-compartment lung model as shown in Figure 2.1. In this model, the lungs are represented as a single lung unit with nonlinear compliance $c(x)$ connected to a pressure source by an airway unit with resistance (to airflow) of R . At time $t=0$, a driving pressure $P_{in}(t)$ is applied to the opening of the parent airway, where $P_{in}(t)$ is generated by the respiratory muscles or a mechanical ventilator. This pressure is applied over the time interval $0 \leq t \leq T_{in}$, which is the inspiratory part of the breathing cycle. At time $t = T_{in}$, the applied airway pressure is released and expiration takes place passively, that is the external pressure $P_{ex}(t)$ is only the atmospheric pressure during the time interval where T_{ex} is the duration of expiration.

The state equation for inspiration (inflation of lung) is given by

$$R_{ex} \dot{x}(t) + \frac{1}{c_{ex}(x)} x(t) = p_{ex}(t), \quad x(T_{in}) = x_0^{ex},$$

$$T_{in} \leq t \leq T_{in} + T_{ex},$$
(2)

2.3 State Equations for a Two Compartment Lung Model

The two-compartment model is of the form shown in Eq. (2) for a single-compartment model where x is the state variable representing volume. The only difference between this model and that of a two-compartment model, is that x in the model for the latter is a state vector with two components, one for each compartment. Note that two sets of state equations are required to completely describe the two-compartment lung model: one for the inspiratory phase, and one for the expiratory phase. Also note that the pressure p appearing in the equations is a forcing function. Solving the state equations gives solutions for the compartmental volumes x for a given input pressure. The solutions do not provide compartmental pressures. The only way to obtain pressure is via the compliance c .

The following points are important:

- The equations are non-linear by the compliance c is a function of the volume x .

Furthermore, the compliance functions for inspiration and expiration are not the same.

- The values of R , the airway resistance, are also different for inspiration and expiration.
- Because of the non-linearity, the system does not have a transfer function, and therefore does not have an inverse.

The significance of the above is that, first, simulation of the model is not trivial because of the need to switch between two models during simulation. The inspiration model is used during the inspiration phase. At the end of this phase the final volume reached in the solution becomes the initial value for the expiration model used in the expiration phase. Second, the non-linearity implies that adaptive inverse dynamics control could be implemented by using a neural network as a controller. However, it is not the intention with this thesis to design a controller at this level of complexity.

2.4 State equations derived from two compartment lung model.

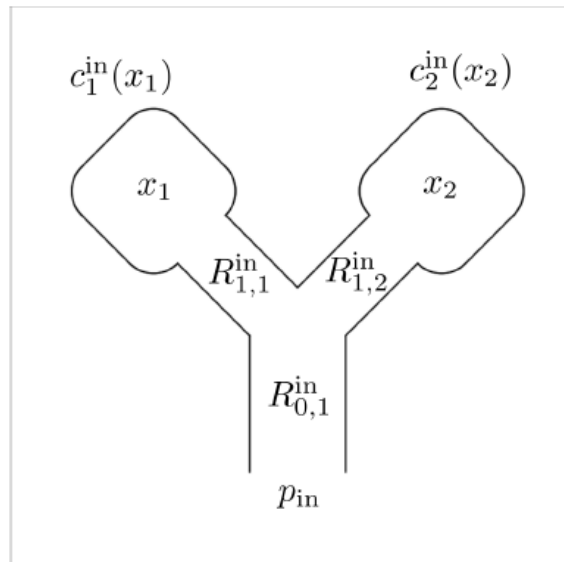


Fig 2.4 State equation two compartment model

Refer to Fig 2.4, the state equations for the two-compartment model can be expressed as

$$\begin{bmatrix} R_{0,1}^{\text{in}} + R_{1,1}^{\text{in}} & R_{0,1}^{\text{in}} \\ R_{0,1}^{\text{in}} & R_{0,1}^{\text{in}} + R_{1,2}^{\text{in}} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{c_1^{\text{in}}(x_1(t))} & 0 \\ 0 & \frac{1}{c_2^{\text{in}}(x_2(t))} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = p_{\text{in}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (3)$$

In compact form (3) can be written as

$$R_{\text{in}} \dot{x}(t) + C_{\text{in}}(x(t)) = p_{\text{in}}(t) \mathbf{e} \quad (4)$$

where $\mathbf{e} = [1,1]^T$ (T means the Transpose)

State equations for the expiration can be expressed as

$$\begin{bmatrix} R_{0,1}^{\text{ex}} + R_{1,1}^{\text{ex}} & R_{0,1}^{\text{ex}} \\ R_{0,1}^{\text{ex}} & R_{0,1}^{\text{ex}} + R_{1,2}^{\text{ex}} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{c_1^{\text{ex}}(x_1(t))} & 0 \\ 0 & \frac{1}{c_2^{\text{ex}}(x_2(t))} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = p_{\text{ex}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (4)$$

which is equivalent to

$$R_{\text{ex}} \dot{x}(t) + C_{\text{ex}}(x(t)) = p_{\text{ex}}(t) \mathbf{e} \quad (5)$$

Equations (3) and (5) above could be confusing since they are written in scalar notation instead of vector-matrix notation. They are given here using the correct notation to be consistent with common practice.

For inspiration in summary, the equations for a two-compartment model can be shown as

$$\mathbf{R}_{\text{in}} \dot{\mathbf{x}}_{\text{in}} + \mathbf{c}_{\text{in}} \mathbf{x}_{\text{in}} = p_{\text{in}} \mathbf{e} \quad (6)$$

$$\dot{\mathbf{x}}_{\text{in}} = -\mathbf{R}_{\text{in}}^{-1} \mathbf{c}_{\text{in}} \mathbf{x}_{\text{in}} + \mathbf{R}_{\text{in}}^{-1} p_{\text{in}} \mathbf{e} \quad (7)$$

and for expiration

$$\mathbf{R}_{ex}\dot{\mathbf{x}}_{ex} + \mathbf{c}_{ex}\mathbf{x}_{ex} = p_{ex}\mathbf{e} \quad (8)$$

$$\dot{\mathbf{x}}_{ex} = -\mathbf{R}_{ex}^{-1}\mathbf{c}_{ex}\mathbf{x}_{ex} + \mathbf{R}_{ex}^{-1}p_{ex}\mathbf{e} \quad (9)$$

It is important to keep in mind that the pressure $P_{in}(t)$ is not a vector quantity. To see why this is so, refer to Fig. 2.4. There is only one input air passage, and this is where the external pressure is applied, either naturally by expansion of the chest, or by means of a mechanical respirator. There is no way to control the pressure individually in each lung airway to the compartments 1 and 2, unless by surgical intervention. However, it is necessary to provide pressure in vector form to facilitate matrix and vector multiplication. That is why it is necessary to introduce the vector $\mathbf{e} = [1 \ 1]^T$. But the two pressure components are identical.

2.5 Matlab Simulations

The above equations are state space equations with the state variables x_1 , x_2 being the compartment volumes. They do not provide compartment pressures. It should be obvious from the notation that

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \text{ and } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2.5.1 Simulation

The main MATLAB program is named lung_model_aic_v0_1.m. There are two functions called by the program

1. The function which sets up the right-hand side of the system of ODEs, lungodeRHS.m.
2. The function which updates the output pressure $p_{in}(t)$, update_Controller.m.

2.5.2 Simulation variables

The airway resistances and compliance polynomial coefficients are in the file named `simvariables.mat`. The variable names are self-explanatory. The air resistance default values may be used as they are, but should it be required to change the values, the new values must be defined in the MATLAB workspace.

2.5.3 Simulation time and simulation time step

The simulation uses a fixed time step of 10ms. The total simulation time is the sum of all time steps. The time of a breathing cycle is also fixed at 5s, 3s for inspiration and 2s for expiration. The desired input pattern

Figure 2.5 shows an example of a valid input pattern. When setting up new patterns, the following should be observed:

- The number of data points in the vector should be exactly 501.
- The first half of the vector, i.e., the inspiration part, must range over 1 to 250.
- The second half of the vector, i.e., the expiration part, must range over 251 to 501.

When the program is started, it prompts the user for the number of breathing cycles to be simulated. This value is then used by the program to join (concatenate) the input pattern repeatedly until it is equal in length to the result of the simulation. This allows the input pattern to be plotted over the model output for comparison purposes. Note that the output vector is a column vector with two elements, `x1` and `x2`. These elements are added row-wise to obtain the total lung volume at each time step.

Chapter 3

Adaptive Inverse Dynamics Control

We know that the two-compartment nonlinear system dynamics have the structure $R \dot{x}_m + C x_m = p$, where p is our system input vector, so select $p = Ra + C x_m$. This is the step we need to take to cancel the system nonlinearities. We don't know the system dynamics perfectly, so we estimate them and make the input behave per $p = \hat{R}a + \hat{C} x_m$, where \hat{R} is the estimate of R and \hat{C} is the estimate of C . Subject to the constraints on the input vector and the rate of convergence of the adaptation law, we know that when the estimate of the system dynamics is close to the true system dynamics, the closed-loop system will behave as $\dot{x}_m = a$. The rest of the derivation shows how to choose an a and an inverse adaptation scheme for our unique problem.

3.1 Derivation

Matrix notation is assumed throughout unless otherwise noted. Dimensions will be commented on for clarity in context.

$$R \dot{x}_m + C x_m = p \quad (10)$$

Consider a nonlinear dynamical robotic system described as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \quad (11)$$

where q is the $n \times 1$ vector of robot joint coordinates, τ is the $n \times 1$ vector of applied joint torques (or forces). $D(q)$ is the $n \times n$ symmetric positive definite inertia matrix, $C(q, \dot{q})\dot{q}$ is the $n \times 1$ vector of centrifugal and Coriolis torques, and $g(q)$ is the $n \times 1$ vector of gravitational torques.

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = Y(q, \dot{q}, \ddot{q})p \quad (12)$$

where $Y(q, \dot{q}, \ddot{q})$ is an $n \times m$ matrix of known functions, known as the regressor and $\hat{P} = [\hat{P}_1, \hat{P}_2, \dots, \hat{P}_m]^T$ is an m -dimensional vector of parameters.

$$\tau = D(q)a + C(q, \dot{q})\dot{q} + g(q) \quad (13)$$

By substituting $\ddot{q} = a$, the vector term a can be defined in terms of a given linear compensator K as

$$a = \ddot{q}^d - Ke \quad (14)$$

with the tracking error $e = q - q^d$, where $q^d(t)$ is an n -dimensional vector of desired joint trajectories

$$[s^2 I_n + K(s)]e(s) = 0 \quad (15)$$

where I_n is an $n \times n$ identity matrix.

$$\ddot{e} + K_v \dot{e} + K_p e = 0 \quad (16)$$

If the gain matrices K_v and K_p are chosen as diagonal matrices with positive diagonal elements then the closed-loop system is linear, decoupled, and exponentially stable.

$$\begin{aligned} \tau &= \hat{D}(q)a + \hat{C}(q, \dot{q})\dot{q} + \hat{g}(q) \\ a &= \ddot{q}^d - K_v \dot{e} - K_p e \end{aligned} \quad (17)$$

where \hat{D} , \hat{C} , and \hat{g} are the estimates of D , C , and g , respectively. Assume that \hat{D} , \hat{C} and \hat{g} have the same functional form as D , C , and g with estimated parameters, then $\hat{P}_1, \hat{P}_2, \dots, \hat{P}_m$ then

$$\hat{D}(q)\ddot{q} + \hat{C}(q, \dot{q})\dot{q} + \hat{g}(q) = Y(q, \dot{q}, \ddot{q})\hat{P} \quad (18)$$

where $\hat{P} = [\hat{P}_1, \hat{P}_2, \dots, \hat{P}_m]^T$ is the vector of the estimated parameters. Substituting (17) into (10) gives

$$D\ddot{q} + C\dot{q} + g = \hat{D}(\ddot{q}^d - K_v \dot{e} - K_p e) + \hat{C}\dot{q} + \hat{g} \quad (19)$$

Adding and subtracting $\hat{D}\ddot{q}$ on the left-hand side of (18) and using (19), we obtain

$$\hat{D}(\ddot{e} + K_v \dot{e} + K_p e) = \tilde{D}\ddot{q} + \tilde{C}\dot{q} + \tilde{g} = Y(q, \dot{q}, \ddot{q})\tilde{p} \quad (20)$$

the error dynamics can be written as

$$\ddot{e} + K_v \dot{e} + K_p e = \hat{D}^{-1}Y\tilde{p} = \Phi\tilde{p} \quad (21)$$

The system can further be expressed in the state-space form as

$$\dot{x} = Ax + B\Phi\tilde{p} \quad (22)$$

where

$$A = \begin{pmatrix} 0 & I_n \\ -K_p & -K_v \end{pmatrix}, B = \begin{pmatrix} 0 \\ I_n \end{pmatrix}, x = \begin{pmatrix} e \\ \dot{e} \end{pmatrix} \quad (23)$$

Based on (17) and (16), we choose the update law

$$\dot{\tilde{p}} = -\Gamma^{-1}\Phi^T B^T P x \quad (24)$$

where $\Gamma = \Gamma^T > 0$ and P is the unique, symmetric, positive definite solution to the Lyapunov equation

$$A^T P + P A = -Q \quad (25)$$

Definitions and substitutions

$$R = D(q), \quad \ddot{q} = \dot{x}_m, \quad \dot{q} = x_m, \quad \tau = p^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} := p$$

Note that $g(q)$ for this problem is 0

Note that x_m denotes the model states. x is reserved for later use.

Note that x_m is used instead of separate state vectors x_{in}, x_{ex} , because the lung compartments can only have one physical configuration at any given time. R is the matrix R_{in} or R_{ex} as appropriate for a given time, this generalized R is used for two reasons.

Firstly, in the analysis, the entries in R do not change the derivation in any material way. Secondly, R is effectively a function of time (it is a function of the states which are functions of time). As far as parameter convergence is concerned, this chain of causality doesn't matter because adaptive inverse control does not need any auxiliary measurements to construct a stabilizing controller. Note that

throughout, the appropriate time dependent model underlying R depends on whether the compartment volume is expanding or contracting as follows:

$$R = \begin{bmatrix} (R_{0,1}^{in} + R_{1,1}^{in})h(\dot{x}_{m_{1,2}}) + (R_{0,1}^{ex} + R_{1,1}^{ex})h(-\dot{x}_{m_{1,2}}) & (R_{0,1}^{in})h(\dot{x}_{m_{1,2}}) + (R_{0,1}^{ex})h(-\dot{x}_{m_{1,2}}) \\ (R_{0,1}^{in})h(\dot{x}_{m_{1,2}}) + (R_{0,1}^{ex})h(-\dot{x}_{m_{1,2}}) & (R_{0,1}^{in} + R_{1,2}^{in})h(\dot{x}_{m_{1,2}}) + (R_{0,1}^{ex} + R_{1,2}^{ex})h(-\dot{x}_{m_{1,2}}) \end{bmatrix}$$

Or at a given instance in time, we can simply write as

$$R = \begin{bmatrix} R_{0,1} + R_{1,1} & R_{0,1} \\ R_{0,1} & R_{0,1} + R_{1,2} \end{bmatrix} \quad (27)$$

where h denotes the Heaviside (unit step) function. This function is defined to be zero for non-positive inputs and one for strictly positive inputs. Hence, its use here is simply to switch the appropriate entries of the R matrix depending on when we are in the “inspiration” or “expiration” mode of operation. Note that $h(\dot{x}_m) = 1$ implies $h(-\dot{x}_m) = 0$, and vice versa. Hence, we never have values for R_{in} and R_{ex} mixing in any way. They are mutually exclusive events. The 1,2 subscript indicates that the simulation will look at one lung compartment or the other to see if it is expanding or contracting. Though the rates may differ, the signs for the rate of change of each lung compartment will always be the same. Hence it is immaterial which one we use in the expression. Only the simulation of the system will incorporate this detailed structure. Parameter estimators used in the simulation obviously won't have access to such detailed structure, hence the simplified notation for the generalized R matrix.

Proceeding as in the paper provided, we identify the governing equation for the dynamics of the two-compartment respiratory system to be

$$R \dot{x}_m + Cx_m = p$$

Note that it is required that the lung resistance matrix R is a positive definite (PD) matrix (denoted $R > 0$). Physically, we can see this will always be the case for this system because we know that by the Gershgorin Circle Theorem that $R_{0,1}, R_{1,1}, R_{1,2} > 0$, hence $R_{0,1} + R_{1,1} > R_{0,1}$ and $R_{0,1} + R_{1,2} > R_{0,1}$.

C is similarly defined with switching functions as in the definition for R above. C is where we introduce the compliance function, which is not assumed known throughout this derivation. Note that if the compliance function is perfectly known, this information can be incorporated to get even faster convergence rates for the remaining parameters. Assuming the compliance function is unknown provides a more general adaptive inverse dynamics control solution to the problem under study and eliminates the need to consider nonlinear effects directly. We can then similarly consider the compliance functions which change with the states to also be functions of time, and therefore we can converge to estimates on these in real time as they evolve without worrying about why they evolve that way. The key here is that we can find a representation of the system dynamics which explicitly shows how the dynamics are linear in the parameters. By using the property of linearity in the parameters, Eq.12 can be written as

$$R \dot{x}_m + Cx_m = Y(x_m, \dot{x}_m)\theta \quad (28)$$

After some calculation, $Y(x_m, \dot{x}_m)$ and θ are determined to be

$$Y(x_m, \dot{x}_m) = \begin{bmatrix} \dot{x}_{m_1} + \dot{x}_{m_2} & \dot{x}_{m_1} & 0 & x_{m_1} & 0 \\ \dot{x}_{m_1} + \dot{x}_{m_2} & 0 & \dot{x}_{m_2} & 0 & x_{m_2} \end{bmatrix} \quad (29)$$

$$\theta = \begin{bmatrix} R_{0,1} + R_{1,1} \\ R_{0,1} \\ R_{0,1} + R_{1,2} \\ c_1^{-1} \\ c_2^{-1} \end{bmatrix} \quad (30)$$

$$\begin{aligned}
\text{That is, } R \dot{x}_m + C x_m &= \begin{bmatrix} R_{0,1} + R_{1,1} & R_{0,1} \\ R_{0,1} & R_{0,1} + R_{1,2} \end{bmatrix} \begin{bmatrix} \dot{x}_{m_1} \\ \dot{x}_{m_2} \end{bmatrix} + \begin{bmatrix} \frac{x_{m_1}}{c_1} \\ \frac{x_{m_2}}{c_2} \end{bmatrix} \\
&= \begin{bmatrix} (R_{0,1} + R_{1,1})\dot{x}_{m_1} + (R_{0,1})\dot{x}_{m_2} + \frac{x_{m_1}}{c_1} \\ (R_{0,1})\dot{x}_{m_1} + (R_{0,1} + R_{1,2})\dot{x}_{m_2} + \frac{x_{m_2}}{c_2} \end{bmatrix} = Y(x_m, \dot{x}_m)\theta
\end{aligned} \tag{31}$$

Then pull out the parameters as defined in the vector θ and the derivation for Y is complete. This will be used later when we construct an adaptive inverse dynamics controller that is guaranteed to provide a closed loop system that is stable in the sense of Lyapunov.

Now we begin to consider the controller structure.

In order to cancel out the nonlinearities inherent in this system we can select input pressure as

$$p = R a + C x_m \tag{32}$$

Then it is apparent by comparing the control law above with the system dynamics that

$$a = \dot{x}_m \tag{33}$$

We then want to enforce a specifically to be

$$a := \dot{x}_m^d - K(s)e, \quad e := x_m - x_m^d \tag{34}$$

where x_m^d is the desired compartment volume profile. Note that the constraints on p constrain the set of compartment volumes. Namely, the compartment volumes cannot be driven independently.

We then use the above definition to work through the construction of an appropriate controller form

$\dot{x}_m \neq \dot{x}_m^d$ (these equations are in the Laplace domain)

$$a = \dot{x}_m = \dot{x}_m^d - K(s)e \Rightarrow \dot{x}_m - \dot{x}_m^d + K(s)e = 0 \Rightarrow sI_2 e + K(s)e = 0 \tag{35}$$

It is common practice in the field of control to mix time and frequency domain representations. The meaning should be clear from context that we are using frequency domain terms such as s and $K(s)$ to act on (filter) time domain variables such as e . This may not seem intuitive to the reader, but if we construct a setup in Simulink which uses blocks to define frequency domain filters like $K(s)$ with the time domain variable e as an input, the software will correctly interpret this so as to provide a filtered time domain signal at the output. This notation will continue to be used throughout this derivation due to its convenience and ease of implementation.

Now select

$$K(s) = K_p := k_p I_2 \quad (36)$$

where I_2 is a 2×2 identity matrix and $k_p > 0$.

So finally we have the control law

$$p = R(\dot{x}_m^d - K_p e) + C x_m \quad (37)$$

The above controller shown in Eq. 22 is only valid if we know the values for R and C perfectly at all points in time. Namely, the above controller is technically just an inverse controller. To construct the adaptive inverse dynamics controller, we realize that there are parameters in R and C that we do not know, hence we need to estimate them.

For this reason, we consider the “hat” indication to mean “the estimate of”, which gives us an estimate of the value of each parameter as the system evolves and the parameter updates are performed recursively over time according to an update law which we (the designers) must define. Typically, we want to design a controller with a control structure which is Lyapunov stable.

Therefore, the adaptive inverse dynamics controller will actually be

$$p = \hat{R} a + \hat{C} x_m \quad (38)$$

Let “tilde” mean “the difference between the actual value and the estimated one”. That is, $R - \hat{R} := \tilde{R}$ and $C - \hat{C} = \tilde{C}$. This estimation error is, since we don’t know R and C precisely

$$\tilde{R} \dot{x}_m + \tilde{C} x_m = Y(x_m, \dot{x}_m) \tilde{\theta} = \hat{R}(\dot{e} + K_p e) \quad (39)$$

Note that $\dot{\tilde{\theta}} = \dot{\theta}$. Since the parameter vector is constant.

Now refer to Eq.13, we have

$$\begin{aligned} Y(x_m, \dot{x}_m) \tilde{\theta} &= \hat{R}(\dot{e} + K_p e) \\ \Rightarrow (\dot{e} + K_p e) &= \hat{R}^{-1} Y \tilde{\theta} := \Phi \tilde{\theta} \end{aligned} \quad (40)$$

Now, we can define a generalized state x

$$x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \quad (41)$$

And the associated dynamics system can be written as (‘0’ is a 2x2 0 matrix)

$$\dot{x} = Ax + B\Phi\tilde{\theta}, \quad A := \begin{bmatrix} 0 & I_2 \\ -K_p & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I_2 \end{bmatrix} \quad (42)$$

Based on the Lyapunov theory of stability, we can use this system to define a Lyapunov function which satisfies the Lyapunov stability properties for a proper choice of the parameter update law. In particular

$$\dot{\tilde{\theta}} = -\Gamma^{-1} \Phi^T B^T P x \quad (43)$$

Hence in the implementation, we will *define* the parameter update law to be $\dot{\theta} = \dot{\tilde{\theta}}$. P is the unique PD solution to the Lyapunov equation $A^T P + PA = -Q$, where Q is a 4×4 PD matrix.

The matrix Γ is called the adaptation gain matrix. This matrix must be selected to be positive definite. A simple selection to ensure this matrix is positive definite would be $\Gamma = \text{diag}(\gamma_1, \dots, \gamma_5)$, $\gamma_1, \dots, \gamma_5 > 0$. The adaptation gains are selected to achieve rapid but stable convergence. These essentially allow for tuning of the bandwidth of the

parameter update. In practical applications, it is best to keep these gains small to reduce unnecessary sensitivity to noise. During simulation, these gains will be adjusted to determine values that work well for the system under study. Without a specified cost function, we must appeal to a trial and error method to find a near-optimal selection of the adaptation gains. We can streamline this task by removing several degrees of freedom in the gain matrix. We select $\gamma^{-1} = \gamma_1 = \dots = \gamma_5$ so that $\Gamma = \gamma^{-1}I_5$ and $\Gamma^{-1} = \gamma I_5$

We can pick up a P matrix easily using Matlab for given numeric entries for K_P and Q , selected for performance. For example

```
>> A = [zeros(2,2), eye(2); -[3 0;0 5], zeros(2,2)]

A =

     0     0     1     0
     0     0     0     1
    -3     0     0     0
     0    -5     0     0

>> Q = diag([1 2 3 4])*10^-14;
>> P = lyap(A,Q)

P =

   -16.8885         0   -0.0000         0
         0   -20.1761         0    0.0000
   -0.0000         0  -50.6655         0
         0    0.0000         0 -100.8806
```

So to make the update law we need to choose as explicit as possible, let's look one more time at the equation that is

$$\dot{\hat{\theta}} = -\Gamma^{-1}\Phi^T B^T P x \quad (44)$$

We know this means

$$\dot{\hat{\theta}} = -\Gamma^{-1}\Phi^T B^T P x = -\Gamma^{-1}Y^T (\hat{R}^{-1})^T B^T P x \quad (45)$$

And by inserting the structure that we know, this update law can be written explicitly as

$\dot{\theta}$

$$\begin{aligned}
&= \frac{-\gamma}{(\hat{R}_{0,1} + \hat{R}_{1,1})(\hat{R}_{0,1} + \hat{R}_{1,2}) - \hat{R}_{0,1}^2} \begin{bmatrix} \dot{x}_{m_1} + \dot{x}_{m_2} & \dot{x}_{m_1} + \dot{x}_{m_2} \\ \dot{x}_{m_1} & 0 \\ 0 & \dot{x}_{m_2} \\ x_{m_1} & 0 \\ 0 & x_{m_2} \end{bmatrix} \begin{bmatrix} \hat{R}_{0,1} + \hat{R}_{1,2} & -\hat{R}_{0,1} \\ -\hat{R}_{0,1} & \hat{R}_{0,1} + \hat{R}_{1,1} \end{bmatrix} [0 \ I_2] P x \\
&= \frac{-\gamma}{\hat{R}_{0,1}\hat{R}_{1,1} + \hat{R}_{0,1}\hat{R}_{1,2} + \hat{R}_{1,1}\hat{R}_{1,2}} \begin{bmatrix} \dot{x}_{m_1} + \dot{x}_{m_2} & \dot{x}_{m_1} + \dot{x}_{m_2} \\ \dot{x}_{m_1} & 0 \\ 0 & \dot{x}_{m_2} \\ x_{m_1} & 0 \\ 0 & x_{m_2} \end{bmatrix} \begin{bmatrix} \hat{R}_{0,1} + \hat{R}_{1,2} & -\hat{R}_{0,1} \\ -\hat{R}_{0,1} & \hat{R}_{0,1} + \hat{R}_{1,1} \end{bmatrix} [0 \ I_2] P x \\
&= \frac{-\gamma}{\hat{R}_{0,1}\hat{R}_{1,1} + \hat{R}_{0,1}\hat{R}_{1,2} + \hat{R}_{1,1}\hat{R}_{1,2}} \begin{bmatrix} (\hat{R}_{1,2})(\dot{x}_{m_1} + \dot{x}_{m_2}) & (\hat{R}_{1,1})(\dot{x}_{m_1} + \dot{x}_{m_2}) \\ (\hat{R}_{0,1} + \hat{R}_{1,2})\dot{x}_{m_1} & (-\hat{R}_{0,1})\dot{x}_{m_1} \\ (-\hat{R}_{0,1})\dot{x}_{m_2} & (\hat{R}_{0,1} + \hat{R}_{1,1})\dot{x}_{m_2} \\ (\hat{R}_{0,1} + \hat{R}_{1,2})x_{m_1} & (-\hat{R}_{0,1})x_{m_1} \\ (-\hat{R}_{0,1})x_{m_2} & (\hat{R}_{0,1} + \hat{R}_{1,1})x_{m_2} \end{bmatrix} [0 \ I_2] P x
\end{aligned}$$

(46)

In summary, we have the system dynamics based on parameter estimates:

$$p = \hat{R} a + \hat{C} x_m \quad (47)$$

We have defined a control structure for a as

$$a = \dot{x}_m = \dot{x}_m^d - K_p e \quad (48)$$

And the parameter update law

$$\dot{\theta} = -\Gamma^{-1} \Phi^T B^T P x \quad (49)$$

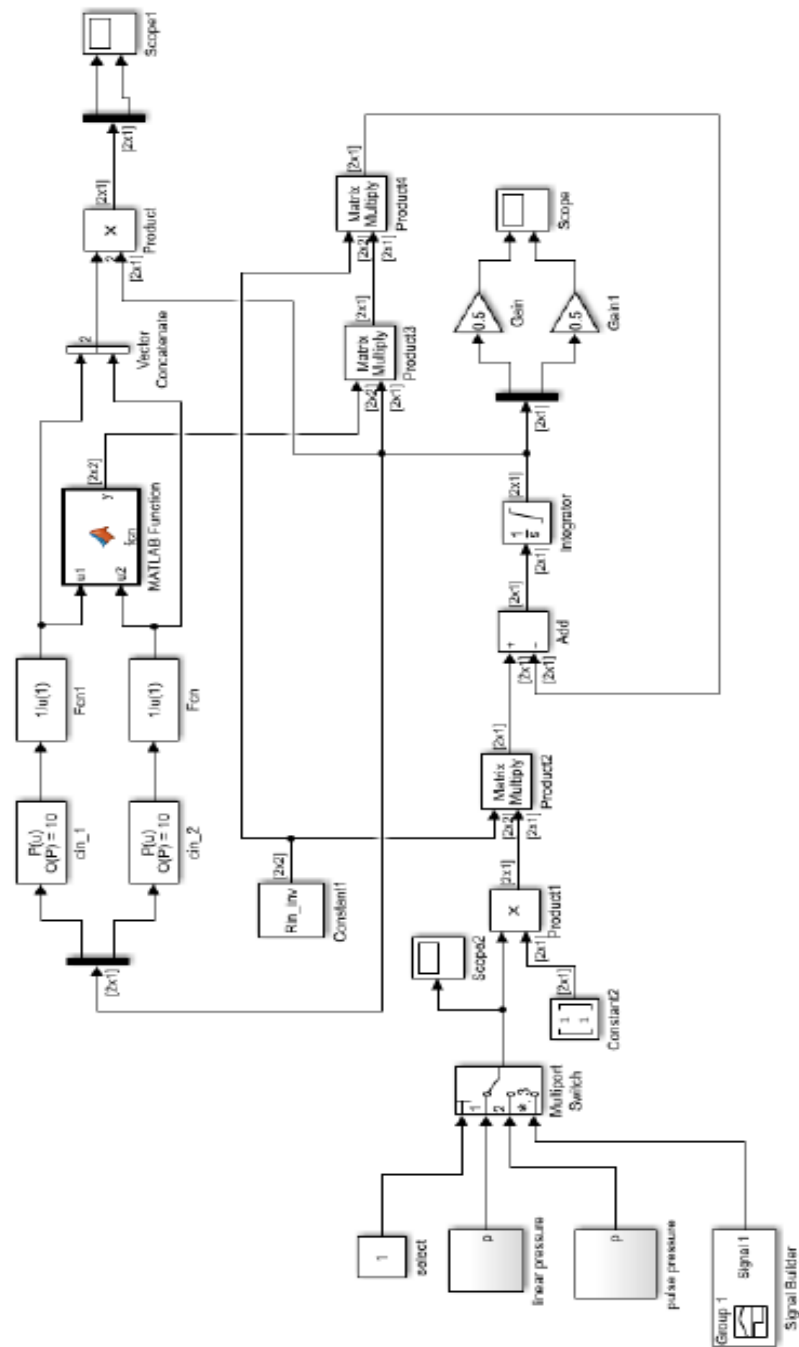
These in combination completely define the adaptive inverse dynamics control for the two-compartment respiratory system.

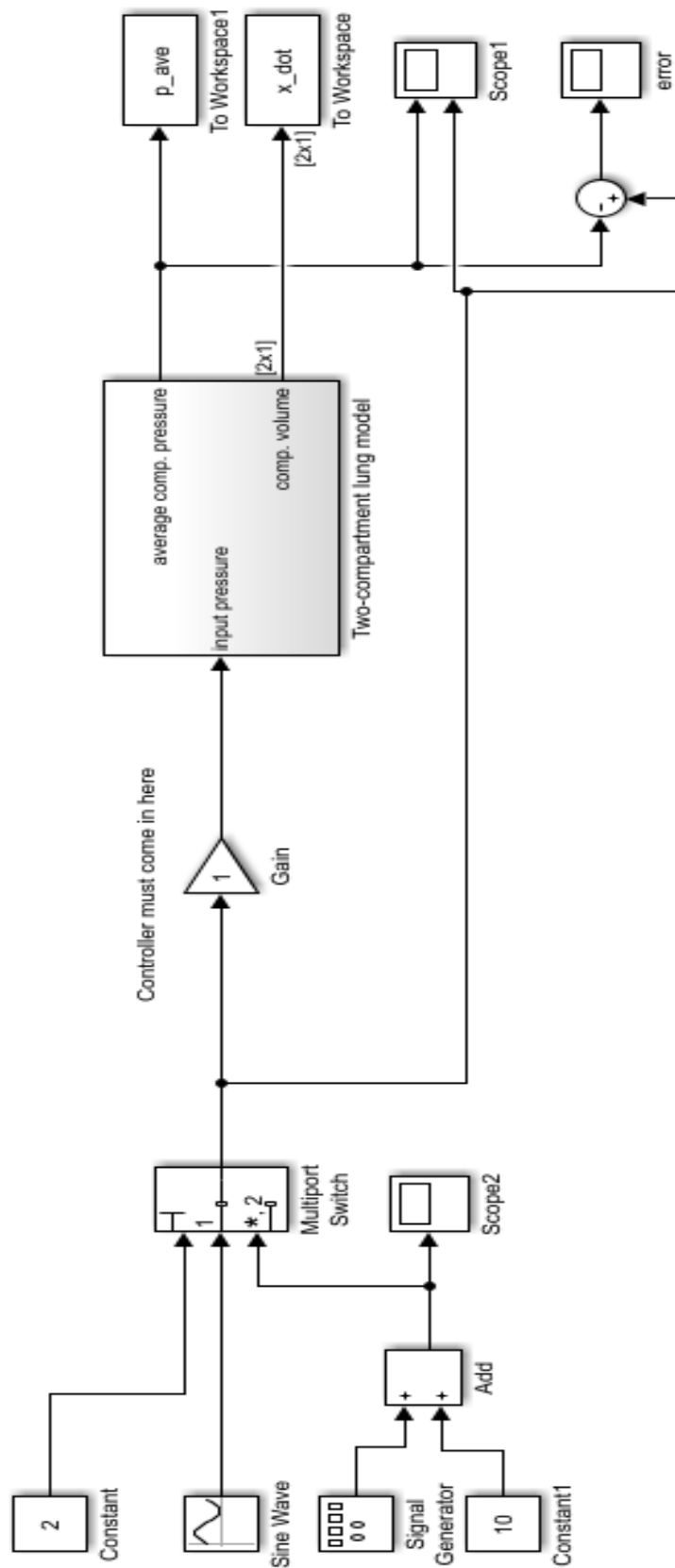
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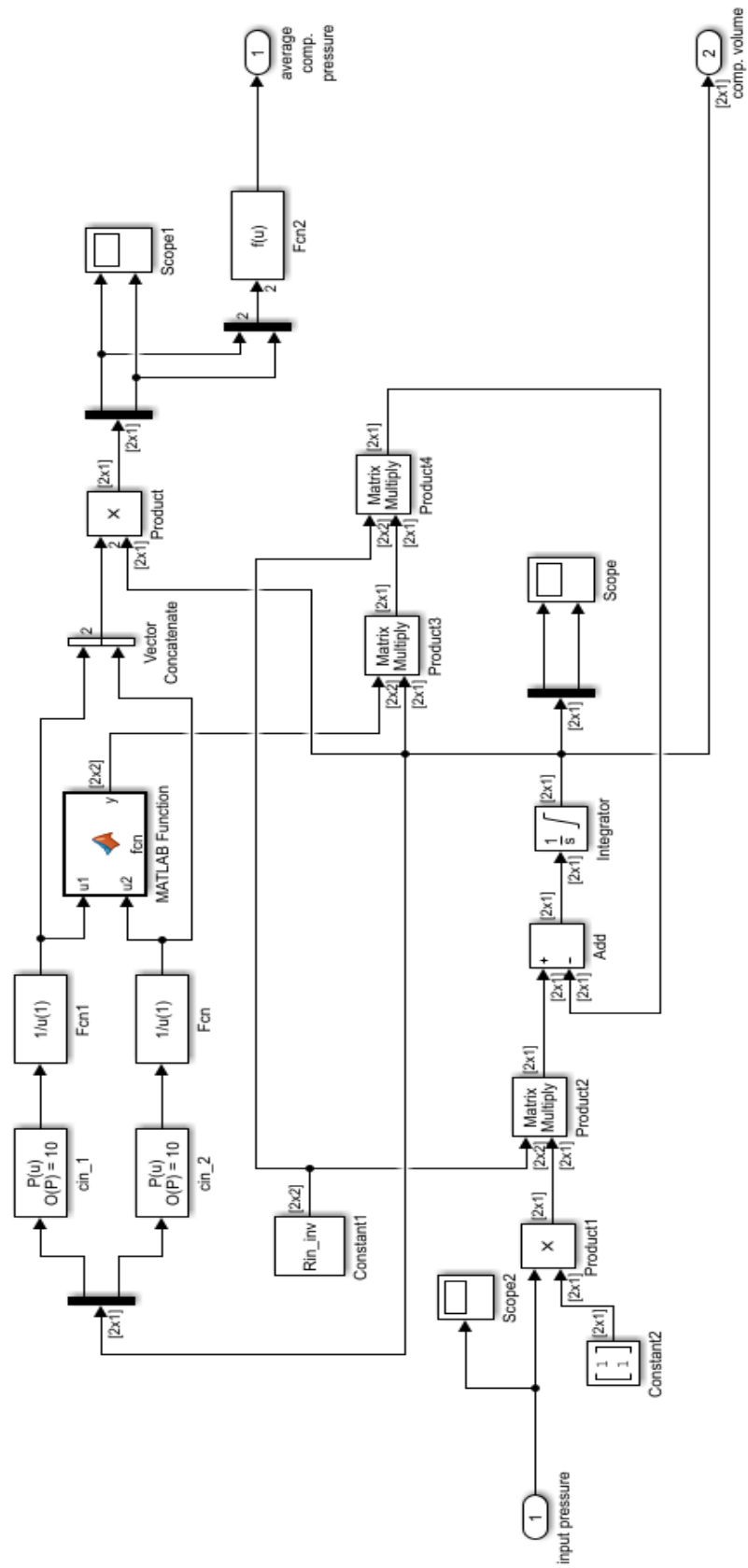
Simulation and Results

It is not necessary to use this model, but it provides an easy way to examine model behavior for various input pressure profiles and changes in parameter values (i.e., airflow resistance and compliance). This model can be either used for inspiration or expiration by merely changing the parameters and integrator initial values. Note that the above equations are state space equations with the state variables x being volume. They DO NOT provide compartment pressures. The only way to obtain pressure is via the non-linear compliance functions.

4.1 Simulink model of the two-compartment lung model



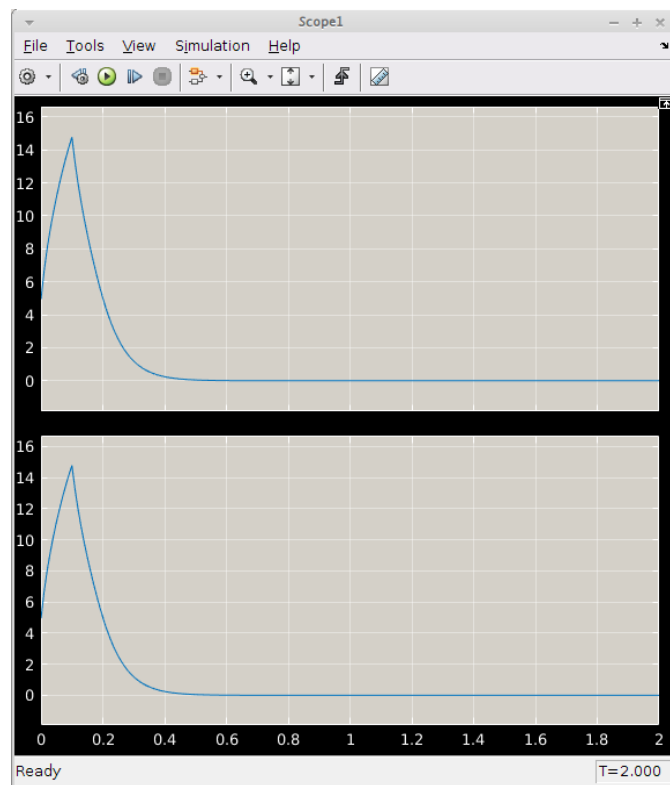
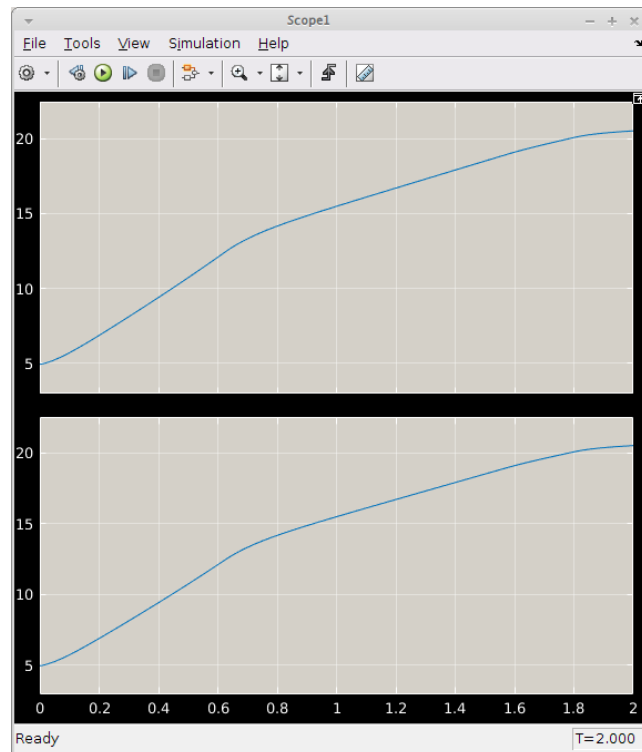


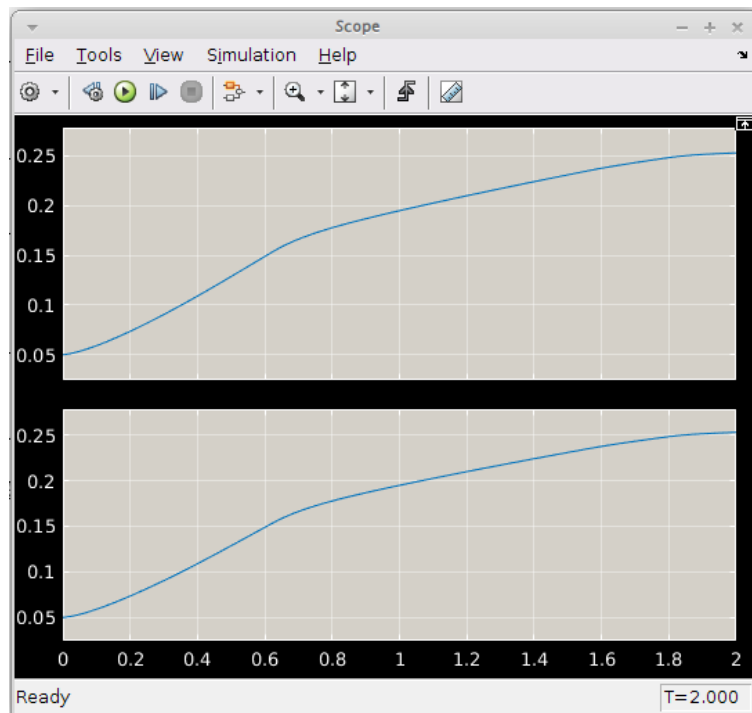
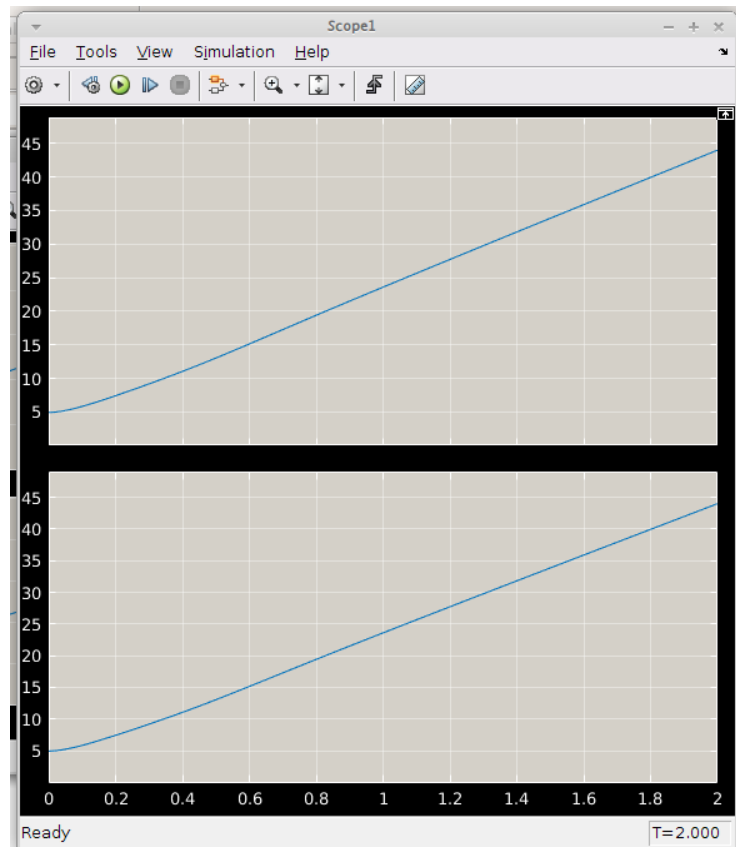


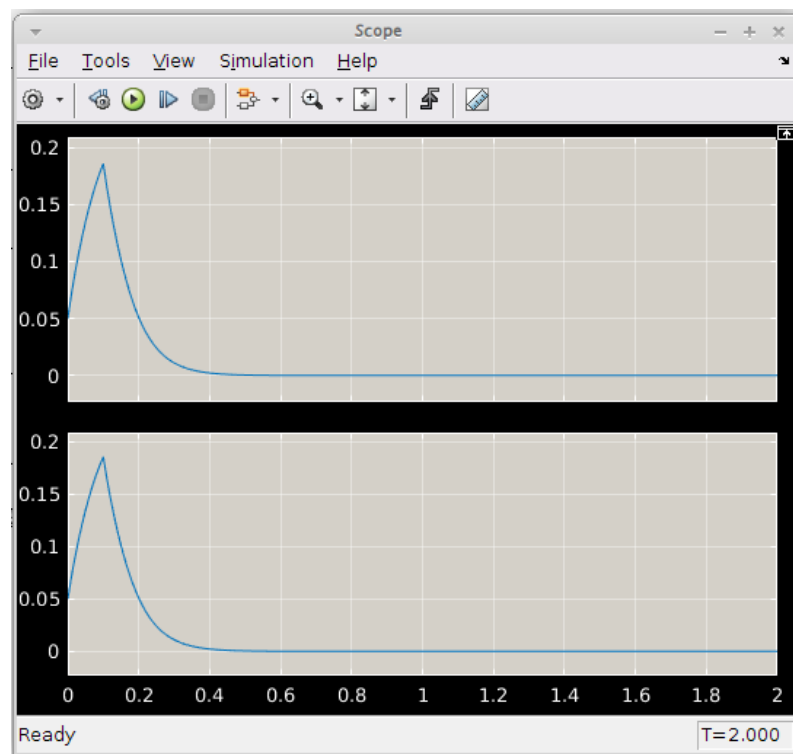
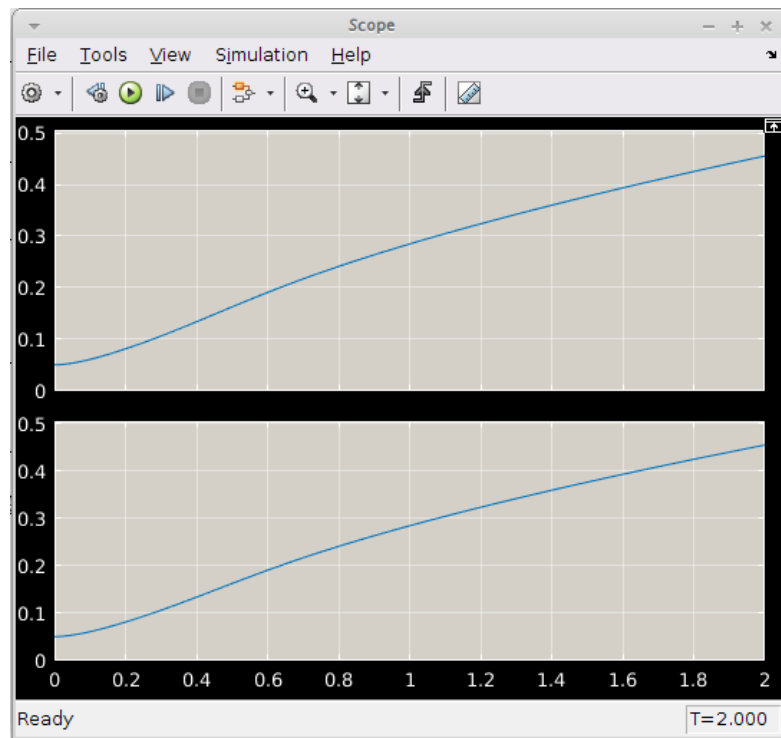
The error is in terms of volume. The input pressure is the output of the controller. The input to the controller is the desired volume as well as the current volume. So we are not controlling the input pressure directly - the controller figures out what input pressure it needs to use to make the output volume match the desired output volume as closely as possible.

Note that the controller is limited to making input pressures of the form $p[1 \ 1]^T$. This means that if the desired volume for compartment 1 is different than the desired volume for compartment 2, or if the R matrix makes it such that the volume in compartment 1 will be greater or lesser than the volume for compartment 2 given equal input pressure, then the controller must decide as to how it will optimally match the actual output volume to the desired output volume

4.2 Simulink SCOPE Graphs:

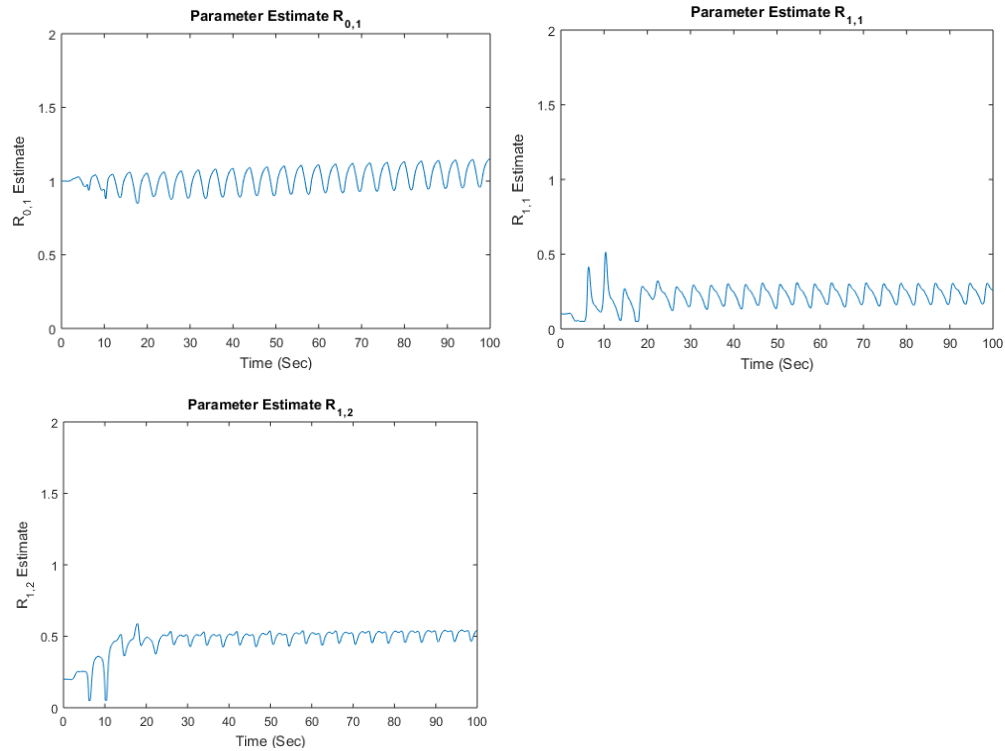


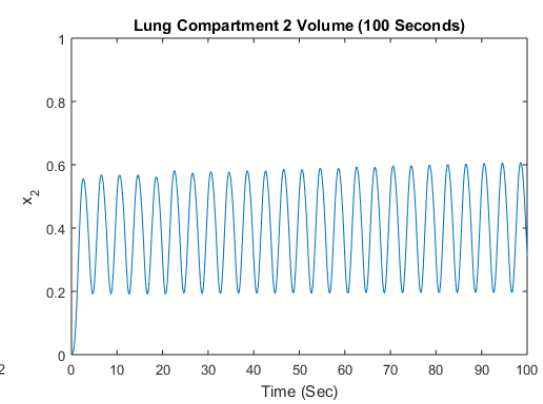
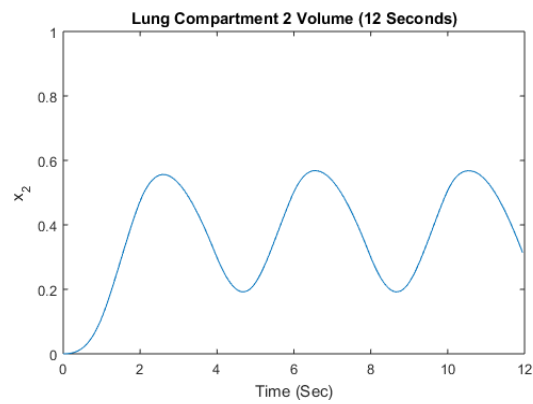
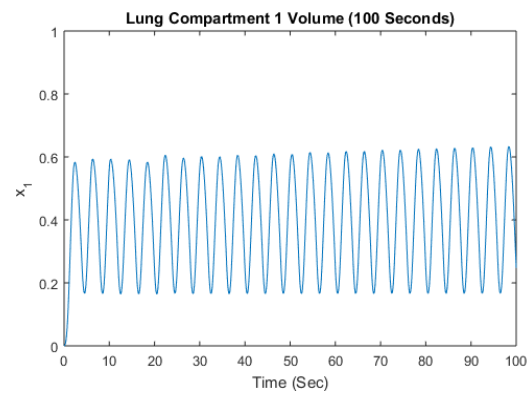
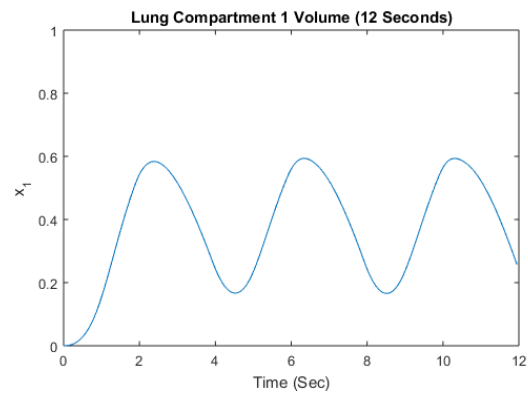
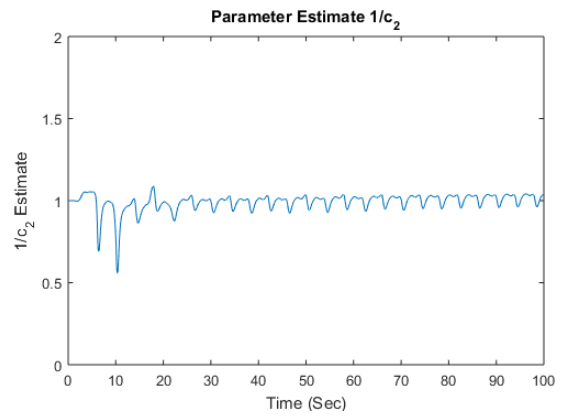
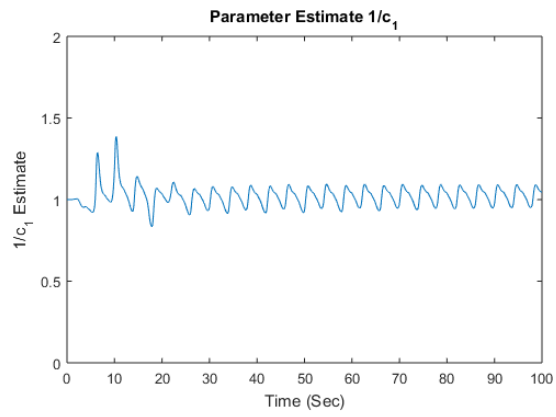


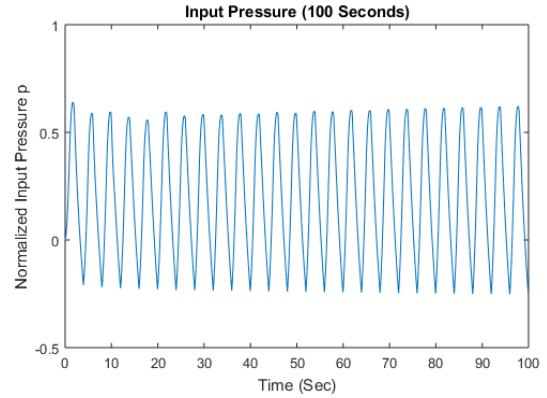
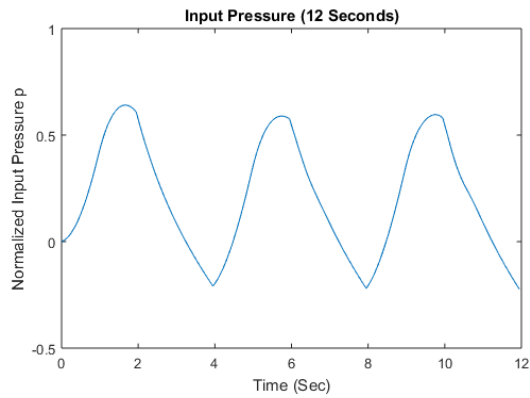
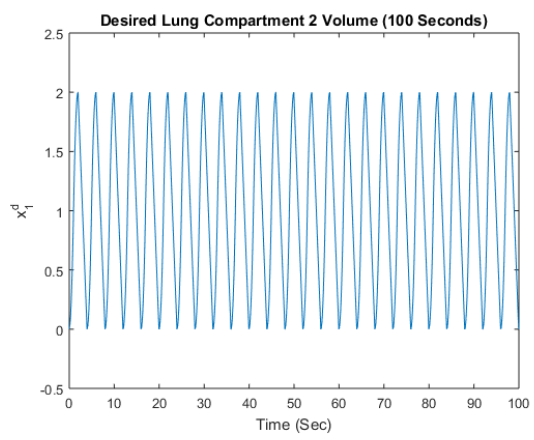
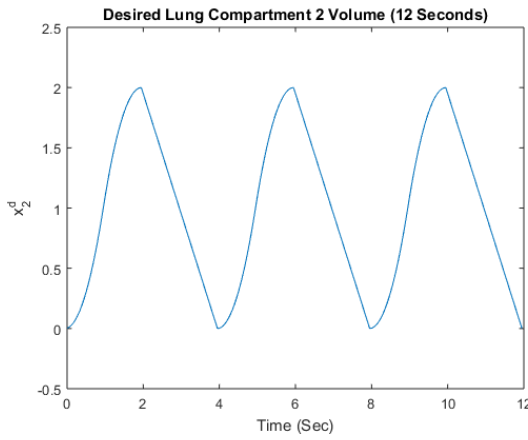
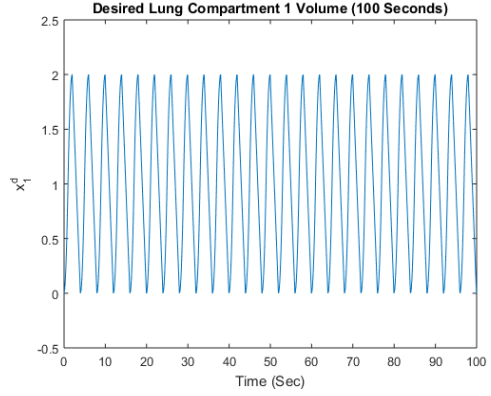
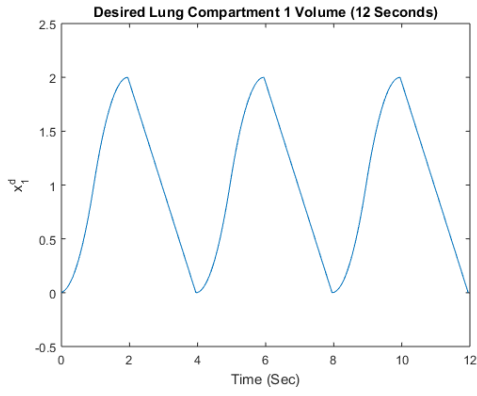


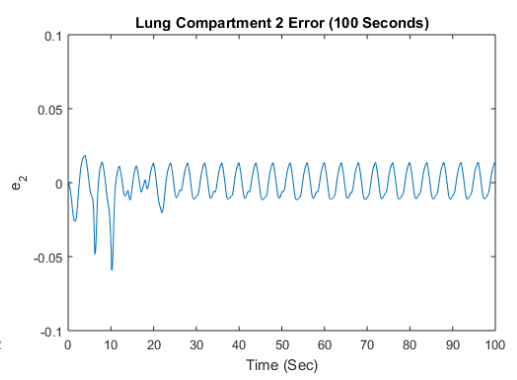
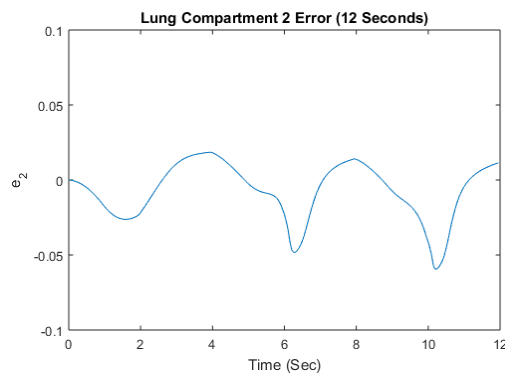
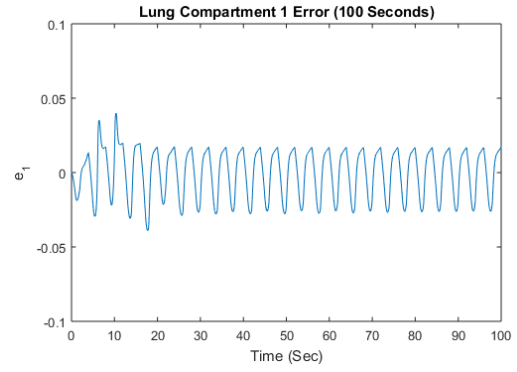
4.3 Graphs

Based on the desired volume pressures and the two-compartment model equation, the adaptive inversed dynamics control scheme is used to control the pressure parameters. The values of the inspiratory and expiratory lung resistance constants and compliances for the two-compartment lung model were taken from [9] and they are: 0,1 in R = 9 cm H₂O/l/s, 1,1 in R = 1,2 in R = 16 cm H₂O/l/s, 0,1 ex R = 18 cm H₂O/l/s, 1,1 ex R = 1,2 ex R = 32 cm H₂O/l/s. The expiratory resistance is assumed two times higher than the inspiratory resistance. The lung compliance is chosen to be 0.1 l/cm H₂O. The inspiration duration time $T_{in} = 2$ s and the expiration time $T_{ex} = 3$ s. The desired air pressures were taken. During the adaptive inverse dynamics control process, the total number of parameters to be estimated is five and Fig. 1 shows the three estimated parameters P_2 , P_4 and P_6 over time during one breathing cycle. Figure 2 shows the tracking errors; for instance, e_2 is the difference between the desired and actual pressures entering the 2nd compartment. Overall, the tracking errors are reasonably small.









Chapter 5

Conclusions

We have applied the adaptive inverse dynamics control method to a two-compartment respiratory system. The implementation of the control scheme consists of a control law and an adaptation law. The control law has the structure of the two-compartment inverse dynamics servo but uses estimates of the dynamics parameters in the computation of pressure applied to the lungs. The adaptation law uses the tracking error to compute the parameter estimates for the control law, stops updating a given parameter when it reaches its known bounds, and resumes updating as soon as the corresponding derivative changes sign. The advantage of using the inverse dynamics control method is that it formulates a globally convergent adaptive controller which does not require approximations such as local linearization, time-invariant, or decoupled dynamics to guarantee the tracking convergence. Simulations show that the tracking errors are acceptably small. The future work includes the robustness study of the control method to the multi-compartment model.

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Adaptive Inverse Control Matlab Code

```

%%%%% (generate .m file or copy and paste into MATLAB workspace) %%%%

clear all

Rin01 = 1;Rin11 = .2;Rin12 = .4;Rex01 = 1;Rex11 = .2;Rex12 = .4;
% define Rin, Rex (these can be changed by any user)

Rin = [Rin01+Rin11,Rin01;Rin01,Rin01+Rin12];Rex =
[Rex01+Rex11,Rex01;Rex01,Rex01+Rex12];iRin=inv(Rin);iRex=inv(Rex); % put terms
in matrix form (do not change)

cin1 = @(xm1) 1/1;cin2 = @(xm2) 1/1;cex1 = @(xm1) 1/1;cex2 = @(xm2) 1/1;
% define Cin, Cex (these are called function handles. user may change the 1/1 term to
any function of the states)

Cin =@(xm1,xm2) [cin1(xm1),0;0,cin2(xm2)];Cex =@(xm1,xm2)
[cex1(xm1),0;0,cex2(xm2)]; % generate a few more function handles
for future use (do not change)

t_end = 100; t_step = .05; %
define fixed simulation step and duration (user may change)

t = [0:t_step:t_end]; siml = length(t); %
generate sim times (do not change)

Q = eye(4); % choose
any positive definite 4x4 square matrix Q (user may change)

Kp = 3*diag([0.9,1.1]); % set
gain matrix (user may change)

P = lyap([zeros(2,2), eye(2); -Kp, zeros(2,2)],-Q)*10^-15;
% calculate P (do not change)

```

```

gam = 100*eye(5); % set
adaptation gain (user may change)

B = [zeros(2,2);eye(2)]; %
define B (do not change)

Y=@(xm1,xm2,xm1_dot,xm2_dot) [(xm1_dot+xm2_dot), (xm1_dot), 0, xm1,
0;(xm1_dot+xm2_dot), 0, (xm2_dot), 0, (xm2)]; % define Y as function handle (do not
change)

base_fun = (t(1:80).^2).*(heaviside(t(1:80))-heaviside(t(1:80)-1)) + (2- ((t(1:80)-
2).^2)).*(heaviside(t(1:80)-1)... % define a base function (repeating pattern) representing
the desired system output (user may change. you may also override on line 20 to use any
nonnegative signal)

    -heaviside(t(1:80)-2)) + (4-t(1:80)).*(heaviside(t(1:80)-2)-heaviside(t(1:80)-4));
%

for init_fun=1:siml % repeat
this pattern for the duration of the simulation (do not change)

fun(init_fun) = base_fun(mod(init_fun,length(base_fun))+1);
%

end %

xm1d = fun;xm2d=fun; %
input tracking waveform (desired compartment volumes. user may change to any
nonnegative function of time.)

lb = 0.05; ub = 2; % define
lower and upper bounds for parameters (user may change, as long as lb = 'lower bound' >
0)

xm1 = 0; xm2 = xm1; theta([1:5],1)=[1,.1,.2,1,1]';theta_dot([1:5],1) =
zeros([5,1]);theta([1:5],2)=theta([1:5],1); % set ICs

theta_dot([1:5],2)=theta_dot([1:5],1); xm1_dot(1) = 0; xm2_dot(1) = xm1_dot; pl=0;
xm1l(1)=0;xm2l(1)=xm1l(1); %

e0([1:2],1) = [0;0]; e0_dot([1:2],1) = [0;0]; x_vec(1,[1:2])=[0,0];
%

Y_filt = zeros([2,5]); %

```

```

for ti = 2:siml % begin
simulation (do not edit anything below this line)

if ((xm1_dot+xm2_dot)>0) %
detect inspiration or expiration mode based on expansion or contraction of lungs and
switch to appropriate coefficients (this allows us to use any input signal without
unnecessary constraints)

R=Rin;iR=iRin; C = Cin(xm1,xm2);
%

else %

R=Rex;iR=iRex; C = Cex(xm1,xm2);
%

end %

R_hat = [(theta(1,ti-1)+theta(2,ti-1)),theta(1,ti-1);theta(1,ti-1),(theta(1,ti-1)+theta(3,ti-
1))]; % generate model estimates based on most recent parameter estimates

C_hat = diag([theta([4:5],ti-1)]); %

lung_sys = c2d(ss(-R\C, iR, eye(2), zeros([2,2])),t_step);
% define lung system at each time step

derivative_filter = c2d((tf([1 .1],[1 10])),t_step);
% use stable derivative and integral implementations (these can only ever be
approximated in software)

integral_filter = c2d((tf([1 10],[1 .1])),t_step); %

est_lung_sys = c2d(ss(-R_hat\C_hat, inv(R_hat), eye(2), zeros([2,2])),t_step);
% define estimated lung system based on model estimates at each time step

p = -0.5*sum(R\C_hat*[xm1;xm2] + R\R_hat*[xm1;xm2]-
R\R_hat*[xm1d(ti);xm2d(ti)]); % input pressure must be of the
form p*[1 1]'. this condition is forced for all combinations of user defined lung volumes.

x_vec(ti,[1:2]) = ((lung_sys.c)*((lung_sys.a)*[xm1;xm2] + (lung_sys.b)*[1;1]*p))'; xm1n
= x_vec(ti,1); xm2n = x_vec(ti,2); % compute actual system output

est_x = (est_lung_sys.c)*((est_lung_sys.a)*[xm1;xm2] + (est_lung_sys.b)*[1;1]*p);
% compute estimated system output

```

```

xm1_dot(ti) = [1 0]*((lung_sys.a)*[xm1;xm2] + (lung_sys.b)*[1;1]*p);
% we could use the derivative filter here but stability is improved by using the actual
derivative from the state equation

xm2_dot(ti) = [0 1]*((est_lung_sys.a)*[xm1;xm2] + (est_lung_sys.b)*[1;1]*p);
%

e0([1:2],ti) = [(xm1-est_x(1));(xm2-est_x(2))];
% compute error

e0_dot([1:2],ti) = -derivative_filter.den{1,1}(2)*e0_dot([1:2],ti-
1)+derivative_filter.num{1,1}(1)*e0([1:2],ti-
1)+derivative_filter.num{1,1}(2)*e0([1:2],ti); % approximate the time
rate of change of error

Y_filt = -
integral_filter.den{1,1}(2).*Y_filt+integral_filter.num{1,1}(1).*(Y(xm1,xm2,xm1_dot(ti)
-1),xm2_dot(ti-
1)))+integral_filter.num{1,1}(2).*(Y(xm1n,xm2n,xm1_dot(ti),xm2_dot(ti))); % use a
Dynamics Parametric Equation (DPM) representation of Y for simulation stability

theta_dot([1:5],ti) = -
(gam\Y_filt*(inv(R_hat))'*B'*P*[e0(1,ti);e0(2,ti);e0_dot(1,ti);e0_dot(2,ti)]);
% implement the estimator per the algorithm provided

theta(:,ti) = min(max(theta(:,ti-1) + theta_dot(:,ti),lb*ones([5,1])),ub*ones([5,1]));
% update the parameter vector using 1D projection based on known upper and lower
bounds

xm1=xm1n;xm2=xm2n;
% xm1n = 'xm1_new', xm1 and xm2 are used in some parts of the code for convenience,
referring to the previous time step's state output)

p_record(ti)=p; %
record the control input signal for plotting purposes

end % end
simulation

figure(1)

plot(t,theta(1,:))

title('Parameter Estimate R_0_1')

```



```

ylabel('R_0,_1 Estimate')

xlabel('Time (Sec)')

set(gcf,'color','white')

figure(2)

plot(t,theta(2,:))

title('Parameter Estimate R_1,_1')

ylabel('R_1,_1 Estimate')

xlabel('Time (Sec)')

set(gcf,'color','white')

figure(3)

plot(t,theta(3,:))

title('Parameter Estimate R_1,_2')

ylabel('R_1,_2 Estimate')

xlabel('Time (Sec)')

set(gcf,'color','white')

figure(4)

plot(t,theta(4,:))

title('Parameter Estimate 1/c_1')

ylabel('1/c_1 Estimate')

xlabel('Time (Sec)')

set(gcf,'color','white')

figure(5)

plot(t,theta(5,:))

title('Parameter Estimate 1/c_2')

ylabel('1/c_2 Estimate')

```

```

xlabel('Time (Sec)')
set(gcf,'color','white')
figure(6)
plot(t(1:240),x_vec(1:240,1))
title('Lung Compartment 1 Volume (12 Seconds)')
ylabel('x_1')
xlabel('Time (Sec)')
set(gcf,'color','white')
figure(7)
plot(t,x_vec(:,1))
title('Lung Compartment 1 Volume (100 Seconds)')
ylabel('x_1')
xlabel('Time (Sec)')
set(gcf,'color','white')
figure(8)
plot(t(1:240),x_vec(1:240,2))
title('Lung Compartment 2 Volume (12 Seconds)')
ylabel('x_2')
xlabel('Time (Sec)')
set(gcf,'color','white')
figure(9)
plot(t,x_vec(:,2))
title('Lung Compartment 2 Volume (100 Seconds)')
ylabel('x_2')
xlabel('Time (Sec)')

```

```

set(gcf,'color','white')

figure(10)

plot(t(1:240),xm1d(1:240))

title('Desired Lung Compartment 1 Volume (12 Seconds)')

ylabel('x_1^d')

xlabel('Time (Sec)')

ylim([-0.5,2.5])

set(gcf,'color','white')

figure(11)

plot(t,xm1d)

title('Desired Lung Compartment 1 Volume (100 Seconds)')

ylabel('x_1^d')

xlabel('Time (Sec)')

ylim([-0.5,2.5])

set(gcf,'color','white')

figure(12)

plot(t(1:240),xm2d(1:240))

title('Desired Lung Compartment 2 Volume (12 Seconds)')

ylabel('x_2^d')

xlabel('Time (Sec)')

ylim([-0.5,2.5])

set(gcf,'color','white')

figure(13)

plot(t,xm2d)

title('Desired Lung Compartment 2 Volume (100 Seconds)')

```

```

ylabel('x_1^d')
xlabel('Time (Sec)')
ylim([-0.5,2.5])
set(gcf,'color','white')
figure(14)
plot(t(1:240),0.5*p_record(1:240))
title('Input Pressure (12 Seconds)')
ylabel('Normalized Input Pressure p')
xlabel('Time (Sec)')
set(gcf,'color','white')
figure(15)
plot(t,0.5*p_record)
title('Input Pressure (100 Seconds)')
ylabel('Normalized Input Pressure p')
xlabel('Time (Sec)')
set(gcf,'color','white')
figure(16)
plot(t(1:240),e0(1,[1:240]))
title('Lung Compartment 1 Error (12 Seconds)')
ylabel('e_1')
xlabel('Time (Sec)')
set(gcf,'color','white')
figure(17)
plot(t,e0(1,:))
title('Lung Compartment 1 Error (100 Seconds)')

```

```

ylabel('e_1')
xlabel('Time (Sec)')
set(gcf,'color','white')
figure(18)
plot(t(1:240),e0(2,[1:240]))
title('Lung Compartment 2 Error (12 Seconds)')
ylabel('e_2')
xlabel('Time (Sec)')
set(gcf,'color','white')
figure(19)
plot(t,e0(2,:))
title('Lung Compartment 2 Error (100 Seconds)')
ylabel('e_2')
xlabel('Time (Sec)')
set(gcf,'color','white')

```